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Optimum parameters for a r.f. power-splitting network

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RESEARCH DEPARTMENT

OPTIMUM PARAMETERS FOR A R.F. POWER-SPLITTING NETWORK

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Section	Title	Page
	Summary	1
1.	Introduction	1
2.	Method	1
3.	Design	2
4.	Better approximations	2

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OPTIMUM PARAMETERS FOR A R.F. POWER-SPLITTING NETWORK

Summary

Splitting networks well matched over a band of frequencies are required for aerial feeder systems. General design formulae are developed and applied to a particular example for purposes of illustration.

1. Introduction

Feeder systems for aerial arrays require networks which share the power between several transmission lines in a given ratio. Such networks should be designed to have uniform response and input impedance over a band of frequencies, and a well known means of achieving this is the network shown in Fig. 1. Each transmission line is a quarter-wave long at some specified centre frequency at which it is not difficult to deduce a relationship between the characteristic impedances of the lines to ensure that the input impedance has the required value. The further requirement for power splitting in a given ratio determines the ratios of the impedances of all but the input transmission line. There still remains one degree of freedom namely, a scaling factor to multiply all the characteristic impedances by. In this report it is shown that the scale factor can be chosen to ensure that the bandwidth of the system is maximum.

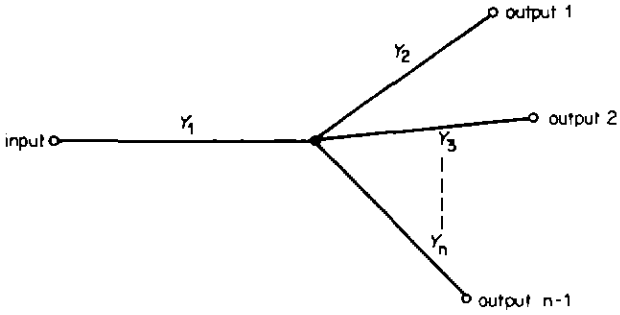

Fig. 1 - Network diagram
Transmission line characteristics: Ch. admittance = 1
Electrical length = λ/4

2. Method

For the purpose of this report it is assumed that each transmission line is loss-less and that the required input and output impedances are all equal. The modifications to the formulae when the latter condition is not met are relatively trivial.

The well-known formula below gives the input admittance of a transmission line of electrical length, θ, characteristic admittance Y_o, terminated by an admittance Y:—

$$Y_{in} = \frac{Y\cos\theta + jY_o\sin\theta}{Y_o\cos\theta + jY\sin\theta} \cdot Y_o \qquad (1)$$

It will be convenient in what follows to write this formula in the alternative form below:—

$$Y_{in} = \frac{Y_o^2}{Y} \cdot \frac{1 - j\cot\theta\,\dfrac{Y}{Y_o}}{1 - j\cot\theta\,\dfrac{Y_o}{Y}} \qquad (2)$$

Assuming a resistive termination, the above function varies with frequency only in the $\cot\theta$ terms. At a frequency at which the line is a quarter-wave $\cot\theta = 0$ and the formula reduces to $Y_{in} = Y_o^2/Y$. Within a band of frequencies centred at the quarter-wave frequency, $\cot\theta$ will vary about zero but will remain within small limits determined by the bandwidth. Hence the following analysis will assume a variable x equal to $\cot\theta$ and formulae based on 'maximally-flat' behaviour at the centre frequency will be deduced.

Returning now to the network of Fig. 1, the lines of characteristic admittance Y_2, Y_3 etc. are each terminated in Y_o. Hence the admittance presented to the line of characteristic admittance Y_1 is Y given by:

$$Y = \sum_{r=2}^{n} \frac{Y_r^2}{Y_o} \cdot \frac{1 - jxY_o/Y_r}{1 - jxY_r/Y_o} \qquad (3)$$

Also the admittance at the input terminal is

$$Y_{in} = \frac{Y_1^2}{Y}\,\frac{1 - jxY/Y_1}{1 - jxY_1/Y} \qquad (4)$$

We require to determine Y_1, Y_2 ... Y_n so that Y_{in} is as nearly equal to Y_o as possible over a range of value of x near zero.

(RA-80)

The next step is to invert Equation (4) so as to express Y in terms of Y_{in}, i.e..

$$Y = \frac{Y_1^2}{Y_{in}} \cdot \frac{1 + jxY_{in}/Y_1}{1 + jxY_1/Y_{in}} \qquad (5)$$

Equations (5) and (3) both give expressions for Y which may be equated:—

$$\frac{Y_1^2}{Y_{in}} \cdot \frac{1 + jxY_{in}/Y_1}{1 + jxY_1/Y_{in}} = \sum_{r=2}^{n} \frac{Y_r^2}{Y_o} \frac{1 - jxY_o/Y_r}{1 - jxY_r/Y_o} \qquad (6)$$

We now put $Y_{in} = Y_o + \epsilon$ where ϵ is (we hope) as small as possible. The left-hand side of Equation (6) may now be written:

$$\frac{Y_1^2}{Y_o} \cdot \frac{1 + jxY_o/Y_1}{1 + jxY_1/Y_o} + \epsilon x \text{ (some function of } Y_1, Y_o \text{ and jx)}$$

and ϵ will be small for small x provided the expression below is small for small x:

$$\frac{Y_1^2}{Y_o} \frac{1 + jxY_o/Y_1}{1 + jxY_1/Y_o} - \sum_{r=2}^{n} \frac{Y_r^2}{Y_o} \frac{1 - jxY_o/Y_r}{1 - jxY_r/Y_o} \qquad (7)$$

Each x-dependent term in expression (7) is of the form:

$$\frac{1 + jx\alpha}{1 + jx/\alpha}$$

Which may be written:

$$1 + jx(\alpha - \frac{1}{\alpha}) + \text{terms of order } (jx)^2 \text{ and higher.}$$

Hence expression (7) will be of order $(jx)^2$ provided that the equations below are true:

$$\frac{Y_1^2}{Y_o} - \sum_{r=2}^{n} \frac{Y_r^2}{Y_o} = 0 \qquad (8)$$

$$\frac{Y_1^2}{Y_o}\left[\frac{Y_1}{Y_o} - \frac{Y_o}{Y_1}\right] + \sum_{r=2}^{n} \frac{Y_r^2}{Y_o}\left[\frac{Y_r}{Y_o} - \frac{Y_o}{Y_r}\right] = 0 \qquad (9)$$

Equation (8), re-written:

$$Y_1^2 = \sum_{r=2}^{n} Y_r^2$$

is the condition for impedance match at the centre frequency. Equation (9) is an extra condition which ensures a maximally-flat response about the centre frequency. Equation (9) may be re-written:

$$\sum_{r=1}^{n} \left[\frac{Y_r}{Y_o} - \frac{Y_r^3}{Y_o^3}\right] = 0 \qquad (10)$$

In the network the ratios $Y_2 : Y_3 : \ldots Y_n$ are determined by the output power ratio, so that we may write $Y_r = \alpha_r Y$ for r = 2 ... n where the α_r^2 are proportional to the output powers and Y is to be determined. The two further conditions give:

$$Y_1 = \alpha_1 Y \text{ where } \alpha_1^2 = \sum_{r=2}^{n} \alpha_r^2 \text{ from Equation 8}$$

$$\text{and} \frac{Y}{Y_o} \sum_{r=1}^{n} \alpha_r = \frac{Y^3}{Y_o^3} \sum_{r=1}^{n} \alpha_r^3 \text{ from Equation 10}$$

and these relationships serve to identify each Y_r uniquely.

3. Design

The following summarises the design method:

(1) Determine $\alpha_2 \ldots \alpha_n$ according to the required power-sharing

(2) Determine $\alpha_1 = \sqrt{\alpha_2^2 + \alpha_3^2 + \ldots \alpha_n^2}$

(3) Determine $Y = Y_o \left\{\frac{\alpha_1 + \alpha_2 + \ldots \alpha_n}{\alpha_1^3 + \alpha_2^3 + \ldots \alpha_n^3}\right\}^{\frac{1}{2}}$

(4) Determine required admittances, $Y_r = \alpha_r Y$

Suppose, for example, it is required to design a network to split the power in the ratio 4 : 1 between two outputs at 50 ohms from an input at 50 ohms.

The characteristic admittances of the three transmission lines are Y_1, Y_2 and Y_3.

From (1) above $\alpha_2^2 = 4$, $\alpha_3^2 = 1$ so that $\alpha_2 = 2$, $\alpha_3 = 1$

From (2) above $\alpha_1 = \sqrt{2^2 + 1^2} = \sqrt{5} = 2{\cdot}236$

$$\text{From (3) above } Y = \frac{1}{50}\left\{\frac{2{\cdot}236 + 2 + 1}{11{\cdot}18 + 8 + 1}\right\}^{\frac{1}{2}} = \frac{{\cdot}509}{50} = $$

$$= {\cdot}0102$$

From (4) above $Y_1 = 2{\cdot}236 \times {\cdot}0102$ mhos

$Y_2 = 2 \times {\cdot}0102$ mhos

$Y_3 = 1 \times {\cdot}0102$ mhos

and, after inverting these numbers, the characteristic impedances are seen to be: Z_1 = 44 ohms, Z_2 = 49 ohms, Z_3 = 98 ohms.

4. Better approximations

The above design formulae were based on the, so-called, 'maximally-flat' response. This approximation places all its emphasis on the centre frequency and does not usually yield the smallest error over the whole of a frequency band. By allowing a small error at the centre frequency it is possible to reduce the error at the band edge.

Our approach to the problem in Section 2 was to approximate the expression (7) by an expression of the form:

$$A + B(jx) + C(jx)^2 + \text{ etc.} \qquad (11)$$

and to equate A and B to zero. The first non-zero term was the $C(jx)^2$ or $-Cx^2$. If x_l is the value of x corresponding to band edge, the error term is related to $-Cx_l^{\,2}$. This can be reduced in magnitude at the expense of increased error at the centre frequency by allowing A to be small and of opposite sign to $-Cx_l^{\,2}$ so that the centre frequency error is now A and the band-edge error $A - Cx_l^{\,2}$.

It would, in principle be possible to solve the pair of equations $B = 0$ and $A - Cx_l^{\,2} = 0$, but, in practice, an equation of degree seven in α_l arises whose solution would clearly be very difficult. An alternative procedure would be to determine the response of a network designed as in Section 3 (using the computer programme 'Frequency analysis of networks' No. TR/14) then make small trial adjustments to Y and recompute the responses until the most suitable had been selected.

Fig. 2 is a Smith Chart plot showing input impedance against frequency for the network, designed earlier, having characteristic impedances of 44 ohms, 49 ohms and 98 ohms. The effect of altering the 44 ohms line to a lower value will, we hope, be to reduce the real part of the input impedance so that it will differ from 50 ohms by a smaller amount at the band edges. A second curve is also drawn on Fig. 2 showing the response of a network with characteristic impedances 43·2 ohms, 49 ohms and 98 ohms. A comparison of the two curves demonstrates an overall improvement in the maximum error over the given frequency band.

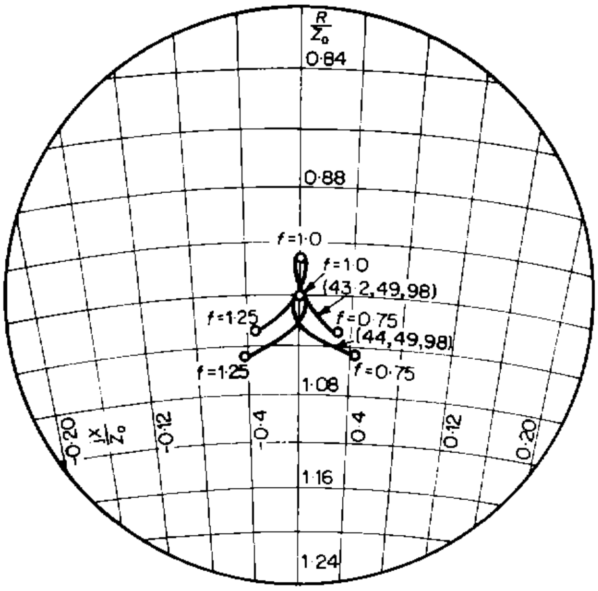

Fig. 2 - Plot of network input impedance vs. frequency